UDC 533.011.8

ON THE DIFFUSION SLIP OF A BINARY GAS MIXTURE

PMM Vol.39, № 3, 1975, pp.442-450

B.M. MARKEEV
(Moscow)

(Received December 18, 1973)

The problem of diffusion slip of a binary mixture of gas is considered on the basis of the Boltzmann equation. Functions of particle distribution with vortices directed toward and away from the surface are assumed different. Solution of the kinematic equation for each of the distribution functions is derived by expanding into series in Hermite-Chebyshev polynomials. A system of transport equations is obtained for an arbitrary law of gas particle interaction. The obtained exact solution of that system is used for the derivation of a formula for the diffusion slip velocity, which is a generalization of the known Kramers-Kistemaker expression for that quantity. The substantial effect of the law of gas particle interaction with a wall on the slip velocity is shown.

The known formula for diffusion slip velocity

$$u_s = \left(\frac{M_\beta - M_\alpha}{y_\alpha M_\alpha + y_\beta M_\beta} - \frac{\sqrt{M_\beta} - \sqrt{M_\alpha}}{y_\alpha \sqrt{M_\alpha} + y_\beta \sqrt{M_\beta}}\right) D_{\alpha\beta} \frac{dy_\alpha}{dx} \tag{0.1}$$

$$M_\alpha = \frac{m_\alpha}{m_\alpha + m_\beta}, \quad y_\alpha = \frac{n_\alpha}{n_\alpha + n_\beta}$$

was derived in [1] by evaluative computations on the assumption of diffusive reflection of particles from the surface. In this formula m_α is the mass of particles of the α kind, n_α is the concentration, and $D_{\alpha\beta}$ is the coefficient of interdiffusion. The diffusion rate determined by this formula is substantially affected by the difference of molecular weights of constituent gases, and is only weakly dependent on the law of interaction between molecules. The additional term in formula (0.1), proportional to the difference of the effective cross sections of particle interaction was obtained in [2] by the method developed in [1]. This term determines the presence of finite slip velocity even in mixtures of gases of the same molecular weight, whenever the laws of interaction are diferent.

Solution of the kinetic equation with the model collision integral had shown [3 – 5], however, that the diffusion slip velocity does not conform to formula (0.1) throughout the region of parameter variation. The question of applicability of this formula remains open.

The method of investigation of diffusion slip of gas based on the kinetic equation with the exact collision integral was developed in [6], where a formula for diffusion slip velocity was derived in the case of diffusion reflection of molecules from the surface with the use of 13-moment approximation. However in the derivation of the distribution function the use was made of the proposition already pointed out by Maxwell [7] that in the surface vicinity the distribution function of molecules impinging on it does not differ from that in the volume of gas. The problem of diffusion slip in the case of arbitrary reflection of molecules from the surface was analyzed in [8], where the diffusion slip velocity was determined by the method of variations and a number of assumptions. The effect of Knudsen layer was taken into account in the formula for the distribution function by an additional term to the function applicable in the volume of gas. In the computation of diffusion velocity this additional term was taken in the form of expansion in moments over the whole velocity space. However, unlike in [6], only the first moment which determines momentum variations, was taken into account, while the momentum flux in the Knudsen layer was neglected. Actually, owing to the collision of molecules impinging on the surface with molecules reflected from it, volume functions are considerably distorted and this results in changes of momentum and momentum fluxes. The extension of results in [6] to the case of an arbitrary reflection of molecules from the surface should be considered the most significant result of [8], while its main shortcoming is the neglect of the momentum flux change in the Knudsen layer.

1. Let us consider a binary gas mixture in a field of partial concentration gradient tangential to the wall. We locate the coordinate origin at the surface with the x-axis normal to it and the z-axis lying on it.

Under stationary flow conditions the distribution function can be determined by the kinetic equation with its right-hand part supplemented by boundary conditions in the form of concentrated sources

$$\mathbf{v}\frac{\partial}{\partial \mathbf{r}}f = J + v_x f(x=0)\,\delta(x) \tag{1.1}$$

$$J = \int dv_1\, d\Omega\, \sigma(w, \vartheta)\, w\, \{f'f_1' - ff_1\}$$

where J is the integral of elastic collisions, $w = v - v_1$ is the relative velocity of colliding particles, ϑ is the angle of dispersion, $\sigma(w, \vartheta)$ is the differential cross section of dispersion, and $\delta(x)$ is the delta-function. We distinguish the molecules which move toward the surface from those leaving it, i.e. $f_\alpha(\mathbf{v}, \mathbf{r}) = f_\alpha^+(\mathbf{v}, \mathbf{r})$ for $v_x > 0$ and $f_\alpha(\mathbf{v}, \mathbf{r}) = f_\alpha^-(\mathbf{v}, \mathbf{r})$ for $v_x < 0$. We represent each of these functions in the form of series in Hermite-Chebyshev polynomials

$$f_\alpha^{\pm}(\mathbf{r}, \mathbf{v}, t) = \frac{n_\alpha}{\pi^{3/2} v_{T\alpha}^3} \exp\left\{-\frac{(v - u_{\pm\alpha z})^2}{v_{T\alpha}^2}\right\} \left\{1 - 2\frac{\sigma_{\pm\alpha xz}}{2n_\alpha T} \times \right. \tag{1.2}$$

$$\left. \frac{m_\alpha}{T}(v_z - u_{\pm\alpha z})\, v_{\pm\alpha x} - \frac{q_{\pm\alpha z}(v_z - u_{\pm\alpha z})}{n_\alpha T}\left[1 - \frac{m_\alpha (v - u_{\pm\alpha z})^2}{5T}\right]\right\}, \quad v_{T\alpha} = \left(\frac{T}{2m_\alpha}\right)^{1/2}$$

where T is the temperature expressed in energy units, $u_{\pm\alpha z}$ is the mean velocity, ; $q_{\pm\alpha z}$ is the heat flux, $v_{T\alpha}$ is the thermal speed of particles, and $\sigma_{\pm\alpha xz}$ is the tensor of viscous stresses. The transport equations for $u_{\pm\alpha z}$, $\sigma_{\pm\alpha xz}$ and $q_{\pm\alpha z}$ can be represented in the form (see Appendix 1)

$$\nabla_x n_\alpha T - \nabla_x \sigma_{\alpha xz}^+ = R_{\alpha z} - \sigma_{0\alpha xz}^+ \delta(x) + n_{0\alpha} T \delta(x) \tag{1.3}$$

$$^2/_5 \nabla_x q_{\alpha z}^+ + p_\alpha \nabla_x u_{\alpha z}^+ = R_{\alpha xz} + [^2/_5 q_{0\alpha z}^+ + p_\alpha u_{0\alpha z}^+]\,\delta(x)$$

$$^2/_5 \nabla_x \sigma_{\alpha xz}^+ = R_{\alpha zkk} - {}^2/_5 \sigma_{0\alpha xz}^+ \delta(x)$$

$$\nabla_x \sigma_{\alpha xz}^- = \nu_{\alpha\beta}^{(1)} n_\alpha m_\alpha u_{\alpha z}^- + \frac{\nu_{\alpha\beta}^{(2)}}{v_{T\alpha}^2} q_{\alpha z}^- + \sigma_{0\alpha xz}^- \delta(x)$$

$$^2/_5 \nabla_x q_\alpha^- + n_\alpha T \nabla_x u_{\alpha z}^- = \sigma_{\alpha xz} \nu_{\alpha\beta}^{(3)} + [^2/_5 q_{0\alpha z}^- + p_\alpha u_{0\alpha z}^-]\,\delta(x)$$

$$\frac{T}{m_\alpha}\nabla_x\sigma_{\alpha xz}^- = \frac{1}{2}n_\alpha m_\alpha v_{T\alpha}^2 \nu_{\alpha\beta}^{(4)} u_{\alpha z}^- + \nu_{\alpha\alpha}^{(5)} q_{\alpha z}^- + \frac{T}{m_\alpha}\sigma_{0\alpha xz}^-\delta(x)$$

$$u_{\alpha z}^\pm = {}^1/_2(u_{\alpha z} \pm u_{-\alpha z}), \quad \sigma_{\alpha xz}^\pm = {}^1/_2(\sigma_{\alpha xz} \pm \sigma_{-\alpha xz}), \; q_{\alpha z}^\pm = {}^1/_2(q_{\alpha z} \pm q_{-\alpha z})$$

The system of transport equations (1.3), unlike in [6. 7], defines the behavior of the binary gas mixture with allowance for discontinuity of the distribution function produced by the effect of the surface on the system. As implied by the problem symmetry, by the method of derivation of system (1.3), and by the form of the latter, it is sufficient to take into consideration only $u_{\alpha z}^\pm$, $\sigma_{\alpha xz}^\pm$ and $q_{\alpha z}^\pm$, since all others are quantities of higher order with respect to parameter $M = u_{\alpha z}T^{1/2}m^{-1/2} \ll 1$. Temperature variation produced by the Joule heat in the Knudsen layer at $M \ll 1$ can also be neglected in the problem.

In deriving (1.3) the expansion of the distribution function was carried out close to partial Maxwellian values, which in the presence of viscous transport of momentum yields, as shown in [9], a more detailed definition of the behavior of a gas mixture. At distances from the surface greater than the mean free path the quantities $u_{\alpha z}^+$, $\sigma_{\alpha xz}^+$ and $q_{\alpha z}^+$ assume, owing to particle collisions, the values $u_{\alpha z}$, $\sigma_{\alpha xz}$ and $q_{\alpha z}$ which define the mixture in its volume. The quantities $u_{\alpha z}^-$, $\sigma_{\alpha xz}^-$ and $q_{\alpha z}^-$ which determine the discontinuity of distribution functions away from the surface, tend to zero, as will be shown below, so that in the velocity space the distribution function and its derivatives become continuous. Hence at considerable distances from the surface system (1.3) converts to the corresponding system of equations of the 13-moment approximation [10].

It should be stressed that the method used for deriving system (1.3) is more flexible than the method used in [11], because away from the surface the distribution function has no discontinuity. The analysis of system (1.3) and the comparison of its solution with known solutions in the case of conventional thermal slip of a simple gas show a good accuracy of the method used here [12].

Analytic expressions for the coefficients $R_{\alpha z}$, $R_{\alpha xz}$, $R_{\alpha zkk}$, $\nu_{\alpha\beta}^{(1)}$, $\nu_{\alpha\beta}^{(2)}$, $\nu_{\alpha\beta}^{(3)}$, $\nu_{\alpha\beta}^{(4)}$ and $\nu_{\alpha\alpha}^{(5)}$ obtained with the use of the Chapman-Cowling integrals are given in Appendix 1.

2. Thus the problem of diffusion slip reduces to the solution of system (1.3). Using the two-way Laplace transformation, we obtain the following solution of this system:

$$u_{\alpha z}^- = (\nu_{\alpha\beta}^{(3)}\mu_\alpha^*)^{-1}[{}^2/_5 q_{0\alpha z}^- + p_\alpha u_{0\alpha z}^-]\operatorname{ch}\lambda_\alpha^{-1}x + \sigma_{0\alpha xz}^-\lambda_\alpha\mu_\alpha^{*-1}x \tag{2.1}$$

$$\begin{gathered} u_0 = \rho^{-1}(\rho_\alpha u_\alpha^+ + \rho_\beta u_\beta^+) = \mu^{-1}\{\mu_\alpha u_{0\alpha}^+ + \mu_\beta u_{0\beta}^+ - (\omega_\alpha - \omega_\beta)(u_{0\alpha}^+ - u_{0\beta}^+)\} + \\ (\sigma_{0\alpha xz}^+ + \sigma_{0\beta xz}^+)x + a(u_\alpha^+ - u_\beta^+) \end{gathered} \tag{2.2}$$

$$\begin{gathered} (u_\alpha{}^+ - u_\beta{}^-) = \frac{D_{\alpha\beta}}{y_\alpha y_\beta}\{a_{n\alpha}\nabla y_\alpha(-1 + \operatorname{ch}\lambda^{-1}x) + \\ (u_{0\alpha}^+ - u_{0\beta}^+)\operatorname{ch}\lambda^{-1}x + (p\mu\lambda)^{-1}(\mu_\beta\sigma_{0\alpha xz}^+ - \mu_\alpha\sigma_{0\beta xz}^+)\operatorname{sh}\lambda^{-1}x \end{gathered} \tag{2.3}$$

$$a = \mu^{-1}[(\mu_\beta\rho_\alpha - \mu_\alpha\rho_\beta)\rho^{-1} + (\omega_\alpha - \omega_\beta)]$$

where $u_{0\pm\alpha z}$, $\sigma_{0\pm\alpha xz}$ and $q_{0\pm\alpha z}$ are limit values of velocity, of the viscous stress tensor, and of the heat flux, respectively. Expressions for λ_α, $\mu_\alpha{}^*$, λ, ω_α and $a_{n\alpha}$ are given in Appendix 2.

Let at some distance from the surface the system reach a stationary state defined by the diffusion slip velocity of gas relative to the surface. It will be seen that the limit values $u_{0\pm\alpha z}, \sigma_{0\pm\alpha xz}$ and $q_{0\pm\alpha z}$ are related by three conditions. The first condition which stipulates the decay of perturbations induced by the surface is of the form

$$\frac{D_{\alpha\beta}}{y_\alpha y_\beta} a_{n\alpha}\nabla y_\alpha + (u_\alpha^+ - u_\beta^+) + (p\lambda\mu)^{-1}(\mu_\beta\sigma_{0\alpha xz}^+ - \mu_\alpha\sigma_{0\beta xz}^+) = 0 \qquad (2.4)$$

The second condition determines the absence of a velocity gradient in the volume of gas

$$(\nabla_x u_0)_\infty = \mu^{-1}(\sigma_{0\alpha xz}^+ + \sigma_{0\beta xz}^+) = 0 \qquad (2.5)$$

The last condition which expresses the limit partial viscosity in terms of limit partial velocity is the consequence of the law of partial reflection of molecules from the surface (*)

$$\begin{aligned}
&\sigma_{0\alpha xz}^+ = W_\alpha^{-1}\{u_{y\alpha} + u_{00} + (u_{0\alpha z}^- - u_{0\beta z}^+)(\gamma_\alpha + \rho_\beta\rho^{-1}) \qquad (2.6)\\
&W_\alpha^{-1} = n_\alpha T(\nu_{\alpha\beta}^{(3)}\lambda_\alpha)^{-1}\varepsilon_\alpha^2(2-\varepsilon_\alpha)^{-2}, \qquad \gamma_\alpha = \gamma(M_\beta - M_\alpha)\\
&\gamma = {}^2/_5(n^2 m_{\alpha\beta}|b|)^{-1}\{{}^2/_5\xi_{\alpha\beta}^2(n^2 m_{\alpha\beta})^{-1} + \xi_{\alpha\beta}b_{\alpha\beta} -\\
&\quad {}^4/_{25}n_\alpha n_\beta(n[D_{\alpha\beta}]_1)^{-1}\xi_{\alpha\beta}n^{-2}\}\\
&u_{y\alpha} = {}^4/_{25}T^{-2}y_\alpha\nabla y_\alpha(b_{\beta\beta} + b_{\alpha\beta}), \qquad u_{00} = \rho^{-1}(\rho_\alpha u_{0\alpha}^+ + \rho_\beta u_{0\beta}^+)
\end{aligned}$$

Formulas for $\xi_{\alpha\beta}$ and $b_{\alpha\beta}$ are given in Appendix 1.

With the obtained formulas (2.4) – (2.6) which interrelate the limit values $\sigma_{0\alpha xz}^\pm$, $u_{0\alpha z}^\pm$ and $q_{0\alpha}^\pm$, it is possible to derive from (2.2) the diffusion slip velocity (for $x \to \infty$). We have

$$\begin{aligned}
&u = u_{0\infty} = -W_\alpha W_\beta(W_\alpha + W_\beta)^{-1}(u_{y\alpha}W_\alpha{}^{-1} + u_{y\beta}W_\beta{}^{-1}) + \qquad (2.7)\\
&\quad[(\alpha - a)(u_{y\alpha} - u_{y\beta}) - aD_{\alpha\beta}(y_\alpha y_\beta)^{-1}a_{n\alpha}\nabla y_\alpha](1 + \Delta_*{}^{-1})^{-1} -\\
&\quad\alpha D_{\alpha\beta}(y_\alpha y_\beta)^{-1}a_{n\alpha}\nabla y_\alpha(1 + \Delta_*)^{-1}\\
&\Delta_* = D_{\alpha\beta}(y_\alpha y_\beta)^{-1}(\lambda p)^{-1}(W_\alpha + W_\beta)^{-1}, \quad \alpha = -\{\gamma_\alpha +\\
&\quad\rho^{-1}(\rho_\beta W_\alpha{}^{-1} - \rho_\alpha W_\beta{}^{-1})(W_\alpha{}^{-1} + W_\beta{}^{-1})^{-1}\}
\end{aligned}$$

3. Formula (2.7) is the most general definition of diffusion rate known in literature (see, e. g., [1 – 5]). It extends the corresponding formulas derived in [1 – 5] to the case of arbitrary interaction of particles between themselves and with the wall. The contribution of terms in (2.7) to the expression for diffusion velocity are different.

To illustrate this aspect let us turn to system (1.3) considered earlier. In fact, in deriving equations of transport by the method of moments, the allowance for the heat flux derivative in the second of Eqs. (1.3) corresponds to the total second approximation in the Chapman-Enskog method [13]. On the other hand, the presence in the fourth of Eqs. (1.3) of the tensor of viscous stresses corresponds to the third approximation. The first two terms in (2.7), as implied by the proof, are due to the taking into account in the fourth of Eqs. (1.3) of the viscous stress tensor, and must appear only in the third approximation in the Chapman-Enskog method. The last term does not, however, vanish

---

*) The law of partial reflection of molecules from a wall is of the form

$$f_\alpha{}^+(v_0, 0, z) = \varepsilon_\alpha f_\alpha{}^0(v, z) + (1 - \varepsilon_\alpha) f_\alpha{}^-(-v_x, v_y, v_z, 0, z)$$

where f_α is the Maxwellian partial distribution function and ε_α is the partial accommodation coefficient.

even in the second approximation, hence its analysis is of considerable interest. Let us represent it in the following form:

$$\gamma_\alpha D_{\alpha\beta} a_{n\alpha} (\nabla y_\alpha)(y_\alpha y_\beta)^{-1}(1 + \Delta_*)^{-1} + u_s \tag{3.1}$$

where only u_s corresponds to the second approximation in the Chapman-Enskog method, while the first term is absent in that approximation. It is, consequently, possible to represent the diffusion slip velocity in the following form:

$$\begin{aligned} u = u_s = &\sqrt{W_\alpha{}^* W_\beta{}^*}\, A \{(W_\alpha^{*-1} - W_\beta^{*-1}) \sqrt{M_\alpha M_\beta} (y_\alpha M_\alpha + y_\beta M_\beta)^{-1} + \\ &{}^1/_2(W_\alpha^{*-1} + W_\beta^{*-1}) [(M_\beta - M_\alpha)(y_\alpha M_\alpha + y_\beta M_\beta)^{-1} - \\ &\delta (\sqrt{M_\beta} - \sqrt{M_\alpha})(y_\alpha \sqrt{M_\alpha} + y_\beta \sqrt{M_\beta})^{-1}]\} (D_{\alpha\beta} a_{n\alpha} \nabla y_\alpha)(1 + \Delta_*)^{-1} \end{aligned} \tag{3.2}$$

$$A = \sqrt{W_\alpha{}^* W_\beta{}^*} (\sqrt{M_\alpha} + \sqrt{M_\beta}) [2 (y_\alpha \sqrt{M_\alpha} W_\alpha{}^* + y_\beta \sqrt{M_\beta} W_\beta{}^*)]^{-1}$$

$$W_\alpha^{-1} = y_\alpha \sqrt{M_\alpha}\, W_\alpha^{*-1}$$

$$\delta^{-1} = 1 - \sqrt{M_\alpha M_\beta} (\sqrt{M_\beta} - \sqrt{M_\alpha})(y_\beta - y_\alpha)(y_\alpha \sqrt{M_\alpha} + y_\beta \sqrt{M_\beta})$$

The structure of formula (3.2) is similar to that of the expression for diffusion velocity obtained in [1, 2]. In fact, if we set $A = a_{n\alpha} = \delta = 1$ and $W_\alpha{}^* = W_\beta{}^*$, we obtain the known Kramer-Kistemaker expression (0.1). The maximum deviation of $a_{n\alpha}$ and δ from unity does not exceed 10%. In the region of parameters where $y_\alpha = y_\beta$ and $M_\alpha \approx M_\beta$ the quantity A is close to unity. The presence in (3.2) of the coefficient Δ_* is the consequence of taking into account of the dynamics of particle collision between themselves and with the wall.

For an arbitrary interaction between particles themselves and with the surface, the condition $W_\alpha{}^* \approx W_\beta{}^*$ is satisfied approximately. Taking this into consideration, formula (3.2) for $y_\alpha \ll y_\beta$ can be expressed as follows:

$$u_s = A\delta^* \sigma_u D_{\alpha\beta} \nabla y_\alpha, \qquad A = \frac{1}{2}\left[1 + \left(\frac{M_\alpha}{M_\beta}\right)^{1/2}\right] \tag{3.3}$$

$$\delta^* = \left[1 + (y_\alpha M_\alpha + y_\beta M_\beta)\frac{1 - \delta}{\sqrt{M_\alpha M_\beta}}\right](1 + \Delta^*)^{-1} a_{n\alpha},$$

$$\sigma_u = \rho^{-1} n \left[1 - \left(\frac{M_\alpha}{M_\beta}\right)^{1/2}\right]\left(\frac{M_\alpha}{M_\beta}\right)^{1/2}$$

For $y_\alpha y_\beta{}^{-1} \ll 1$, formula (3.3) for diffusion velocity differs from the corresponding expression in [5] which was obtained on the basis of the kinetic equation with the model collision integral and coefficient $A\ \delta^+$ which is equal unity throughout the region of parameter variation.

To compare these results with those in [8] we adduce the formula for σ_u derived there on the basis of Boltzmann's equation

$$\sigma_u = \frac{1}{2}\left[1 - 2\frac{M_\alpha}{M_\beta} + \sqrt{\frac{M_\alpha}{M_\beta}}\right]$$

It will be seen that the formula in [8] differs from (3.3) not only by the absence of coefficient $A\delta^*$, but also by the expression for σ_u. This is because in [8], unlike in this investigation and in [5], the variation of viscous transport of momentum across the Knudsen layer was neglected in the analysis of the diffusion slip in a binary mixture.

Let us consider a mixture of gases of nearly the same masses and similar laws of molecular interaction. In that case $W_\alpha^{*-1} - W_\beta^{*-1}$ is defined by

$$\begin{gathered}(W_\alpha^{*-1} - W_\beta^{*-1}) = W^{*-1}\{[B\,(\sigma_\alpha - \sigma_\beta)(\sigma_\alpha + \sigma_\beta)^{-1} + \qquad (3.4)\\ \frac{1}{2}\,[\varepsilon_\alpha^2(2 - \varepsilon_\alpha)^{-2} + \varepsilon_\beta^2(2 - \varepsilon_\beta)^{-2}]\;\; C\,(M_\beta - M_\alpha) - \\ [\varepsilon_\beta^2(2 - \varepsilon_\beta)^{-2} - \varepsilon_\alpha^2(2 - \varepsilon_\alpha)^{-2}]\} \\ W_\alpha^* = W_\beta^* = W^* \quad \text{for } m_\alpha = m_\beta,\ \sigma_\alpha = \sigma_\beta,\ \varepsilon_\alpha = \varepsilon_\beta = 1\end{gathered}$$

where σ_α is the molecule cross section, and expressions for B and C are given in Appendix 3. Thus the first term in (3.2) for the diffusion velocity with allowance for (3.4) is the consequence of the difference in the laws of interaction of particles of different varieties. In the particular case of reflection of molecules from a wall the corresponding term was obtained by Brock [2]. The second term in (3.4) reflects the systematic allowance for the dynamics of collision of particles between themselves. Of the greatest interest is the third term in (3.4) which is due to the difference in the laws of particle interaction with the surface. It determines the appearance of finite diffusion slip velocity in a binary mixture of gases with mechanically equivalent molecules but subject to different laws of molecule interaction with a wall.

It should be pointed out that formula (2.7) which is a general expression for the magnitude of diffusion slip, represents one of the fundamental results of the present investigation. It is derived on the basis of Boltzmann's equation and differs from the corresponding formula in [8] by the systematic taking into account not only of the dynamics of particle collision in the Knudsen layer, but also of the momentum flux in that layer.
In the second approximation the quantity of diffusion slip extends the formulas of [1 – 5] to the case of arbitrary interaction between particles themselves and with the surface.
In the related parameter region formula (3.2) yields, as previously indicated, the results obtained in [1, 2].

As previously indicated, the allowance for the heat flux gradient in the second of Eqs. (1.3) in the derivation of equations of transport (1.3) corresponds to the complete second approximation in the Chapman-Enskog method, while the allowance for the derivative of the viscous stress tensor in the fourth of Eqs. (1.3) corresponds to the third approximation. System (1.3) is characterized by two length scales: the hydrodynamic scale related to gradients of macroscopic quantities and applied along the surface, and the scale determined by the decrease of the wall effect which is proportional to the mean free path. Since the transport equation (1.3) represents a linear system with respect to the unknown functions, it is possible to maintain that the contribution of successive approximations related to gradients of hydrodynamic quantities along the surface is small in comparison with the terms that are taken into account [14]. However, this cannot be said about the gradients of $u_{\pm\alpha z}$, $\sigma_{\pm\alpha xz}$ and $q_{\pm\alpha z}$ across the Knudsen layer, since the ratio of the mean free path to the characteristic scale of variation of these quantities is far from small. Owing to the approximation of distribution functions by a limited number of terms, the effect of gradients of indicated quantities across the Knudsen layer in subsequent approximations remains obscure. The latter is the main shortcoming of the method of moments. One of the effective means of its elimination is apparently provided by the analysis of boundary conditions of slip with the use of the method of matching external and internal asymptotic expansions [15].

In conclusion the author thanks V. V. Struminskii for his interest in this paper.

Appendix 1. Substituting the distribution function (1.2) into the kinetic equation (1.1) and using the theory of parameter-dependent integrals [16], after some simple but cumbersome computations, we obtain the system of transport equations (1.3) in which $\nu^{(1)}_{\alpha\beta}$, $\nu^{(2)}_{\alpha\beta}$, $\nu^{(3)}_{\alpha\beta}$, $\nu^{(4)}_{\alpha\beta}$, $\nu^{(5)}_{\alpha\alpha}$, $R_{\alpha z}$, $R_{\alpha xz}$ and $R_{\alpha zkk}$ are defined as follows:

$$\nu^{(1)}_{\alpha\beta} = \nu^{(1)} + \delta\nu^{(1)} + \delta\nu^{(1)}_{\alpha}(M_\beta - M_\alpha), \qquad \delta\nu^{(1)}_{\alpha} = \Omega^{(1,1)}_{\alpha\beta} + 2\Omega^{*(1,1)}_{\alpha\beta}$$

$$\nu^{(1)} = \sum_{\substack{\alpha,\beta \\ \alpha\neq\beta}} [\Omega^{(1,1)}_{\alpha\alpha} + \Omega^{(1,1)}_{\beta\alpha} + 2(\Omega^{*(1,1)}_{\alpha\alpha} + \Omega^{*(1,1)}_{\beta\alpha})]$$

$$\delta\nu^{(1)} = \sum_{\substack{\alpha,\beta \\ \alpha\neq\beta}} [\Omega^{(1,1)}_{\alpha\alpha} - \Omega^{(1,1)}_{\beta\alpha} + 2(\Omega^{*(1,1)}_{\alpha\alpha} - \Omega^{(1,1)}_{\beta\alpha})]$$

$$\nu^{(2)}_{\alpha\beta} = -8\{{}^1/_4(3\Omega^{*(0,0)}_{\alpha\alpha} - 2\Omega^{*(0,1)}_{\alpha\alpha}) - M_\alpha M_\beta(3\Omega^{*(0,0)}_{\alpha\beta} - 2\Omega^{*(0,1)}_{\alpha\beta})\} +$$
$$2\{(\Omega^{(1,1)}_{\alpha\alpha} + 2\Omega^{*(1,1)}_{\alpha\alpha}) - 2/5(\Omega^{(1,2)}_{\alpha\alpha} + 2\Omega^{*(1,2)}_{\alpha\alpha})\} +$$
$$8M^2_\beta\{(\Omega^{(1,1)}_{\alpha\beta} + 2\Omega^{*(1,1)}_{\alpha\beta}) - {}^2/_5(\Omega^{(1,2)}_{\alpha\beta} + 2\Omega^{*(1,2)}_{\alpha\beta}\}$$

$$\nu^{(3)}_{\alpha\beta} = \nu^{(3)} + \delta\nu^{(3)} + \delta\nu^{(3)}_{\alpha}(M_\beta - M_\alpha) + \delta\nu^{(3)}_{*}(M_\beta - M_\alpha)$$

$$\nu^{(3)} = \sum_{\alpha,\beta}\left\{2\left[\frac{3}{4}\Omega^{*(0,0)}_{\alpha\beta} - \frac{1}{10}\Omega^{*(0,2)}_{\alpha\beta}\right] + \right.$$
$$\left.[\Omega^{(1,1)}_{\beta\alpha} + 2\Omega^{*(1,1)}_{\beta\alpha}] + \frac{3}{10}[\Omega^{(2,2)}_{\beta\alpha} + 2\Omega^{*(2,2)}_{\beta\alpha}]\right\}$$

$$\delta\nu^{(3)} = \sum_{\alpha,\beta}\left\{\left[\frac{3}{2}(\Omega^{*(0,0)}_{\alpha\alpha} - \Omega^{*(0,0)}_{\beta\alpha}) - \frac{1}{10}(\Omega^{*(0,2)}_{\alpha\alpha} - \Omega^{*(0,2)}_{\beta\alpha})\right] + \right.$$
$$\left.\frac{3}{10}[(\Omega^{(2,2)}_{\alpha\alpha} + 2\Omega^{*(2,2)}_{\alpha\alpha}) - (\Omega^{(2,2)}_{\beta\alpha} + 2\Omega^{*(2,2)}_{\beta\alpha})]\right\}$$

$$\delta\nu^{(3)}_{\alpha} = -4\{({}^3/_2\Omega^{*(0,0)}_{\alpha\beta} + {}^1/_5\Omega^{*(0,2)}_{\alpha\beta}) - {}^3/_{10}(\Omega^{(2,2)}_{\alpha\beta} + 2\Omega^{*(2,2)}_{\alpha\beta})\}$$

$$\delta\nu^{(3)}_{*} = 4({}^3/_4\Omega^{*(0,0)}_{\alpha\beta} - {}^1/_{10}\Omega^{*(0,2)}_{\alpha\beta}) - 2(\Omega^{(1,1)}_{\alpha\beta} + 2\Omega^{*(1,1)}_{\alpha\beta}) + {}^3/_5(\Omega^{(2,2)}_{\alpha\beta} + 2\Omega^{*(2,2)}_{\alpha\beta})$$

$$\nu^{(4)}_{\alpha\beta} = [{}^5/_2(\Omega^{(1,1)}_{\alpha\alpha} + 2\Omega^{*(1,1)}_{\alpha\alpha}) + (\Omega^{(1,2)}_{\alpha\alpha} + 2\Omega^{(1,2)}_{\alpha\alpha})] +$$
$$4M_\beta[{}^5/_2 M_\alpha(\Omega^{(1,1)}_{\alpha\beta} + 2\Omega^{*(1,1)}_{\alpha\beta}) + M_\beta(\Omega^{(1,2)}_{\alpha\beta} + 2\Omega^{*(1,2)}_{\alpha\beta})]$$

$$\nu^{(5)}_{\alpha\alpha} = 2[(\Omega^{(3,4)}_{\alpha\alpha} + \Omega^{(3,4)}_{\alpha\beta}) - (\Omega^{*(3,3)}_{\alpha\alpha} + \Omega^{*(3,3)}_{\alpha\beta}) - {}^5/_4\nu^{(2)}_{\alpha\beta}]$$

$$R_{\alpha z} = \sum_\beta\left\{B^{(1)}_{\alpha\beta}(u_\alpha{}^+ - u_\beta{}^+)_z + B^{(2)}_{\alpha\beta}\left[\frac{\overset{+}{q}_{\alpha z}}{m_\alpha p_\alpha} - \frac{\overset{+}{q}_{\beta z}}{m_\beta p_\beta}\right]\right\}$$

$$R_{\alpha xz} = -\sum_\beta\left\{B^{(3)}_{\alpha\beta}\frac{\overset{+}{\sigma}_{\alpha xz}}{n_\alpha} + B^{(4)}_{\alpha\beta}\frac{\overset{+}{\sigma}_{\beta xz}}{n_\beta}\right\}$$

$$R_{\alpha zkk} = \frac{T}{m_\alpha}\sum_\beta\left\{\frac{5}{2m_\alpha}B^{(2)}_{\alpha\beta}(u_\alpha{}^+ - u_\beta{}^+)_z + B^{(5)}_{\alpha\beta}\frac{\overset{+}{q}_{\alpha z}}{p_\alpha} + B^{(6)}_{\alpha\beta}\frac{\overset{+}{q}_{\beta z}}{p_\beta}\right\}$$

where $m_{\alpha\beta}$ is the reduced mass and

$$B^{(1)}_{\alpha\beta} = -\frac{n_\alpha n_\beta T}{n\,[D_{\alpha\beta}]_1}, \qquad B^{(2)}_{\alpha\beta} = -T\xi_{\alpha\beta}$$

$$B^{(3)}_{\alpha\alpha} + B^{(3)}_{\alpha\beta} + B^{(4)}_{\alpha\alpha} = \frac{p^2}{T}\, a_{\alpha\alpha}, \qquad B^{(4)}_{\alpha\beta} = \frac{p^2}{T}\, a_{\alpha\beta} \quad (\alpha \neq \beta)$$

$$B^{(5)}_{\alpha\alpha} + B^{(5)}_{\alpha\beta} + B^{(6)}_{\alpha\alpha} = -\frac{5}{2}\,\frac{p^2}{T}\, b_{\alpha\alpha}, \qquad B^{(6)}_{\alpha\beta} = -\frac{5}{2}\,\frac{p^2}{T}\, b_{\alpha\beta} \quad (\alpha \neq \beta)$$

$$[D_{\alpha\beta}]_1 = \frac{n_\beta T}{n m_{\alpha\beta}\Omega^{(1,1)}_{\alpha\beta}}, \qquad \xi_{\alpha\beta} = \frac{1}{T}\,\frac{n_\alpha n_\beta T}{n\,[D_{\alpha\beta}]_1}\, m_{\alpha\beta}\left[\frac{6}{5}\, C_{\alpha\beta} - 1\right]$$

$$a_{\alpha\beta} = -2 y_\alpha y_\beta \,\{(m_\alpha + m_\beta)\, n\,[D_{\alpha\beta}]_1\}^{-1}\,(1 - {}^3\!/_5 A_{\alpha\beta})$$

$$a_{\alpha\alpha} = \frac{{y_\alpha}^2}{[\mu_{\alpha\alpha}]_1} + \sum_{\beta\neq\alpha} 2 y_\alpha y_\beta \,\{(m_\alpha + m_\beta)\, n\,[D_{\alpha\beta}]_1\}^{-1} \left(1 + \frac{3}{5}\,\frac{m_\beta}{m_\alpha}\, A_{\alpha\beta}\right)$$

$$b_{\alpha\beta} = -\frac{4}{25}\left(\frac{T}{p}\,\frac{m^2_{\alpha\beta}}{m_\alpha m_\beta}\right)\frac{y_\alpha y_\beta}{[D_{\alpha\beta}]_1}\left\{\frac{55}{4} - 3B_{\alpha\beta} - 4A_{\alpha\beta}\right\}$$

$$b_{\alpha\alpha} = \frac{{y_\alpha}^2}{[\eta_{\alpha\alpha}]_1} + \sum_{\beta\neq\alpha}\frac{4}{25}\left(\frac{T}{p}\,\frac{m^2_{\alpha\beta}}{m_\alpha m_\beta}\right)\frac{y_\alpha y_\beta}{[D_{\alpha\beta}]_1}\times$$

$$\left\{\frac{15}{2}\,\frac{m_\alpha}{m_\beta} + \frac{25}{4}\,\frac{m_\beta}{m_\alpha} - 3\,\frac{m_\beta}{m_\alpha}\, B_{\alpha\beta} + 4A_{\alpha\beta}\right\}$$

$$A_{\alpha\beta} = \Omega^{(2,2)}_{\alpha\beta}\,(\Omega^{(1,1)}_{\alpha\beta})^{-1}, \qquad B_{\alpha\beta} = (5\Omega^{(1,2)}_{\alpha\beta} - \Omega^{(1,3)}_{\alpha\beta})\,(3\Omega^{(1,1)}_{\alpha\beta})^{-1}$$

$$C_{\alpha\beta} = \Omega^{(1,2)}_{\alpha\beta}\,(3\Omega^{(1,1)}_{\alpha\beta})^{-1}$$

$$[\mu_{\alpha\alpha}]_1 = \frac{10}{3}\,\frac{n_\alpha T}{\Omega^{(1,1)}_{\alpha\alpha}}, \qquad v_{\alpha\beta} = (v^2_{T\alpha} + v^2_{T\beta})^{1/2}, \qquad [\eta_{\alpha\alpha}]_1 = \frac{25}{2}\,\frac{n_\alpha T}{m_\alpha \Omega^{(2,2)}_{\alpha\alpha}}$$

$$\Omega^{(l,s)}_{\alpha\beta} = \frac{8}{3\sqrt{\pi}}\, n_\beta v_{\alpha\beta} \int dg\, d\Omega\sigma\,(w,\,\vartheta)\,(1 - \cos^l\vartheta)\; g^{2s+3}\, e^{-g^2}$$

$$\Omega^{*(l,s)}_{\alpha\beta} = \frac{4}{3\sqrt{\pi}}\, n_\beta v_{\alpha\beta} \int dg\, d\Omega\sigma\,(w,\,\vartheta)\cos^l\vartheta\, g^{2s+3}\, e^{-g^2}$$

Appendix 2. In the derivation of solutions (2.1) – (2.3) we introduce with the use of formulas in Appendix 1 the following notation:

$$\lambda_\alpha^{-2} = \frac{n_\alpha m_\alpha}{{\mu_\alpha}^*}\left(\nu^{(1)}_{\alpha\beta} - \frac{1}{2}\,\nu^{(2)}_{\alpha\beta} S_\alpha\right), \qquad {\mu_\alpha}^* = \frac{n_\alpha T}{\nu^{(3)}_{\alpha\beta}}\left(1 - \frac{2}{5}\, S_\alpha\right)$$

$$S_\alpha = (\nu^{(4)}_{\alpha\beta} - \nu^{(1)}_{\alpha\beta})\,(\nu^{(5)}_{\alpha\alpha} - {}^1\!/_2\nu^{(2)}_{\alpha\beta})^{-1}$$

$$\lambda^2 = (\lambda^2_{\alpha\alpha}\lambda^2_{\beta\beta} - \lambda^2_{\alpha\beta}\lambda^2_{\beta\alpha})\,(\lambda^2_{\alpha\alpha} + \lambda^2_{\beta\beta} + \lambda^2_{\alpha\beta} + \lambda^2_{\beta\alpha})^{-1}$$

$$\lambda^2_{\alpha\alpha} = -\frac{n D_{\alpha\beta}\omega_\alpha}{n_\alpha n_\beta T}, \qquad \lambda^2_{\alpha\beta} = \frac{(\omega_\alpha + \mu_{\alpha\beta})\, n D_{\alpha\beta}}{n_\alpha n_\beta T}, \qquad \mu_{\alpha\beta} = y_\alpha y_\beta\,\frac{|\,a\,|_{\beta\alpha}}{|\,a\,|}$$

$$\omega_\alpha = \frac{2}{5}\left(\frac{T}{p}\right)^2 \sum_\delta \frac{\mu_{\alpha\delta}}{y_\delta}\left\{\frac{1}{m_\alpha}\,\xi^*_{\alpha\beta}\,\frac{\overset{*}{\eta}_{\delta\alpha}}{y_\alpha} - \frac{1}{m_\beta}\,\xi^*_{\beta\alpha}\,\frac{\overset{*}{\eta}_{\delta\beta}}{y_\beta}\right\}$$

$$\overset{*}{\eta}_{\alpha\beta} = y_\alpha y_\beta\,|\,b^*\,|_{\beta\alpha}\,/\,|\,b^*\,|, \qquad D_{\alpha\beta} = [D_{\alpha\beta}]_1\,(1 - \Delta_{\alpha\beta})^{-1}$$

$$a_{n\alpha} = 1 + \frac{2}{5}\left(\frac{T}{p}\right)^2 \xi_{\alpha\beta}\left[\frac{|\,b\,|_{\alpha\alpha}}{m_\alpha\,|\,b\,|} + \frac{|\,b\,|_{\beta\beta}}{m_\beta\,|\,b\,|} - \frac{|\,b\,|_{\alpha\beta}}{m_\beta\,|\,b\,|} - \frac{|\,b\,|_{\beta\alpha}}{m_\alpha\,|\,b\,|}\right]$$

$$\Delta_{\alpha\beta} = \frac{n\,[D_{\alpha\beta}]_1}{n_\alpha n_\beta T}\left(\frac{T}{p}\right)^2 \xi_{\alpha\beta}\left[\frac{T}{m_\alpha}\,\xi^*_{\alpha\beta}\left(\frac{|\,b^*\,|_{\alpha\alpha}}{m_\alpha\,|\,b^*\,|} - \frac{|\,b^*\,|_{\alpha\beta}}{m_\beta\,|\,b^*\,|}\right) + \right.$$

$$\frac{T}{m_\beta}\xi^*_{\beta\alpha}\left(\frac{|b^*|_{\beta\beta}}{m_\beta|b^*|}-\frac{|b^*|}{m_\beta|b^*|}\right)$$

$$\xi^*_{\alpha\beta}=\xi_{\alpha\beta}-\frac{2}{5}\,\frac{m_\alpha}{T}\,\frac{n_\alpha n_\beta T}{n\,[D_{\alpha\beta}]_1}\,,\qquad b^*_{\alpha\beta}=b_{\alpha\beta}+b^{(1)}_{\alpha\beta}$$

$$b^{(1)}_{\alpha\alpha}=-\frac{2}{5}\sum_{\beta\neq\alpha}\frac{\xi_{\alpha\beta}}{n^2m_\alpha}\,,\qquad b^{(1)}_{\alpha\beta}=\frac{2}{5}\,\frac{\xi_{\alpha\beta}}{m_\beta n^2}\quad\text{for}\quad\alpha\neq\beta$$

Appendix 3. In the investigation in the second approximation of the diffusion slip in a binary mixture of gases with mechanically equivalent molecules it is necessary to know the parameters B and C. These are defined by the following expressions:

$$B=\left(\frac{\delta\nu^{(1)}}{\nu^{(1)}}-\frac{\delta\nu^{(4)}}{\nu^{(4)}}\right)\frac{\Omega^{(2,2)}_{\alpha\alpha}+\Omega^{(2,2)}_{\alpha\beta}}{\Omega^{(2,2)}_{\alpha\alpha}-\Omega^{(2,2)}_{\alpha\beta}}\,,$$

$$C=\frac{1}{2}\left[\frac{\delta\nu^{(1)}_\alpha+\delta\nu^{(1)}_\beta}{\nu^{(1)}}-\frac{\delta\nu^{(4)}_\alpha+\delta\nu^{(4)}_\beta}{\nu^{(4)}}\right]$$

To calculate the diffusion velocity of a binary mixture of gases with mechanically equivalent molecules in the third approximation it is necessary to know the following quantities:

$$u_{y\alpha}+u_{y\beta}=(u_{y\alpha}-u_{y\beta})\,\frac{\sigma_\alpha-\sigma_\beta}{\sigma_\alpha+\sigma_\beta}\,,\qquad u_{y\alpha}-u_{y\beta}=\frac{4}{25}\,\frac{y_\alpha y_\beta\nabla y_\alpha}{T\,|b|\,[\eta]_1}$$

REFERENCES

1. Kramers, H. A. and Kistemaker, J., On the slip of a diffusing gas mixture along a wall. Physica, Vol. 10, № 8, 1943.
2. Brock, J. R., Forces on aerosols in gas mixtures. J. Colloid. Sci., Vol. 18, № 2, 1963.
3. Ialamov, Iu. I., Ivchenko, I. N. and Deriagin, B. V., Calculation of diffusion slip velocity of a binary gas mixture. Dokl. Akad. Nauk SSSR, Vol. 180, № 2, 1968.
4. Ivchenko, I. N. and Ialamov, Iu. I., On the diffusion slip of a binary gas mixture. Izv. Akad. Nauk SSSR, MZhG, № 4, 1971.
5. Zharov, V. A., Determination of slip velocity of a binary mixture of gases. Izv. Akad. Nauk SSSR, № 2, 1972.
6. Kucherov, R. Ia. and Rikenglaz, L. E., Slip and temperature jump at the boundary of a gas mixture. ZhETF, Vol. 36, № 16, 1959.
7. Maxwell, J. C., On stresses in rarified gases arising from inequalities of temperature. Phil. Trans. Roy. Soc., London, Vol. 170, pt. 1, 1879.
8. Loyalka, S. K., Velocity slip coefficient and the diffusion slip velocity for multicomponent gas mixture. Phys. Fluids, Vol. 14, № 12, 1971.
9. Markeev, B. M., On the problem of voscous transport of momentum in a gas mixture. Zh. Tekh. Fiz., № 7, 1974.
10. Kolodner, I., Moment Description of Gas Mixtures. New York University, 1957.
11. Bakanov, S. P. and Deriagin, B. V., On the problem of the state of gas moving in the vicinity of a solid surface. Dokl. Akad. Nauk SSSR, Vol. 139, № 1, 1961.

12. Markeev, B. M., On a particular method of investigation of gas slip along a solid surface. Zh. Tekh. Fiz., № 5, 1974.
13. Chapman, S. and Cowling, T., Mathematical Theory of Inhomogeneous Gases. (Russian translation), Izd. Inostr. Lit., Moscow, 1960.
14. Foch, J., Ulenbeck, G. and Losa, M., Theory of sound propagation in mixtures of monatomic gases. Phys. Fluids, Vol. 15, № 7, 1972.
15. Darrozés, J., Les glissements d'un gas parfait sur les parois dans un ecoulement de Couette. Recherches Aerospat., № 119, 1967.
16. Vladimirov, B. S., Equations of Mathematical Physics. (In Russian), "Nauka", Moscow, 1971.

Translated by J. J. D.

UDC 533.6.011

ACCELERATION, COMPRESSION AND STABILITY OF A PLANE LAYER OF MATTER IRRADIATED BY A LASER

PMM Vol. 39, № 3, 1975, pp. 451-457

Iu. V. AFANAS'EV, E. G. GAMALII, O. N. KROKHIN and V. B. ROZANOV
(Moscow)

(Received June 18, 1974)

We consider a simple gasdynamic model of acceleration and compression of a plane material layer irradiated by a laser. We establish the conditions under which a limiting isentropic compression takes place, and investigate its stability. We also consider the problem of transfer of the laser radiation energy to the accelerated layer.

A number of experimental and theoretical investigations (see e.g. [1 – 3]) dealt with the problem of transfer of the mechanical recoil impulse to the material target, the impulse resulting from the evaporation and hydrodynamic scattering of the material acted upon by the laser radiation. It is also known that at sufficiently high radiation flux densities, compression waves and in particular shock waves, appear in the nonvaporized material. It is clear that, if the amount of vaporized mass is comparable with the total mass of the target, then the nonvaporized part can be speeded up to velocities approaching that of the flow of the vaporized matter and, under certain conditions, compressed to the densities exceeding appreciably the density of the normal, condensed state. This effect of accelerating low-mass solid targets is of interest in connection with a general problem of accelerating small particles [4] to velocities of 10^5 to 10^7 cm/sec and higher.

1. Let us consider a plane one-dimensional problem of the action of laser radiation with the flux density of q_0, on a plane layer of condensed matter with the initial mass per unit area equal to M. The process of accelerating the layer is determined by the parameters of the material at the vaporization boundary separating the condensed and the gaseous phases, and for this reason the equations of motion must include the gasdynamic laws of conservation of mass flux, impulse and energy at this boundary.

In accordance with [2], in the present case we have